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ABSTRACT

The role of review in mathematics instruction is briefly summarized in terms of reasons, timing, and types. Promoting retention is the primary reason for reviewing; continuity, assimilation, diagnosis, learning prerequisites, and confidence are also noted. Research clearly indicates that review should be systematically planned and incorporated into the instructional program at regular intervals. Among the types of review discussed (with some specific examples) are outlining, questioning, testing, group size, homework content, difficulty level, and games. Nine references are included. (MNS)

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THE ROLE OF REVIEW IN MATHEMATICS INSTRUCTION

"We'll begin with a review . . ."

How often are those words said? And how frequently is the reaction a negative one? You probably know teachers who show little enthusiasm for review: it seems to be considered a necessary but dull chore. Many students reflect the boredom they've learned to associate with reviewing.

Yet: what does the word mean? Re-view. Look again. See from a new perspective, in time if not in space. The word carries a hidden promise of excitement or potential that we seem to miss.

What are the reasons for reviewing? When should reviewing be done? What instructional procedures for reviewing are effective? Answers to these questions, drawn from research and other literature, are outlined in the sections that follow.

Reasons for Reviewing

Review is valuable because it increases both learning and retention. We know that in many areas of learning there is considerable forgetting over time. For instance, some evidence indicates that approximately two-thirds of the concepts "learned" in high school and college courses are forgotten within two years. The problem is even more crucial in mathematics, where skills and concepts are taught that cannot be regained simply by rereading. Review helps to promote better retention of such skills and concepts.

In addition to promoting retention, there are other, somewhat overlapping reasons why reviewing should be incorporated into mathematics instruction:

(1) Review promotes continuity and helps students to attain a more comprehensive view of the mathematical topics covered. It helps them summarize main ideas, develop generalizations, and get an overall view of what they have been learning piecemeal.

(2) Review helps students to assimilate or consolidate what they have learned, enabling them to fit ideas into new patterns.

(3) Review serves as a diagnostic tool, revealing weaknesses and strengths to students and teachers. It helps them to identify what is already known and what is not yet known; then reteaching can be planned.

(4) Review assures that the prerequisites needed for learning new content have been mastered.

(5) Review adds to students' confidence in their ability to move successfully to new mathematical topics.

Timing of Reviews

Most textbooks incorporate review sections at both the beginning of the year and at the end of most chapters. Saxon (1982) takes exception to this pattern, terming it "spastic." He argues that review should be continuous. Instead of presenting 25 problems on a new topic, he suggests that only three or four be given, along with 25 review problems. His data show that this approach helps students attain better mastery.

Research clearly indicates that review should be systematically planned and incorporated into the instructional program. Before a new topic or unit is begun, an inventory can help the teacher ascertain whether any prerequisite knowledge needed for the new topic is missing. Such a review also helps students to pull together the mathematical ideas they'll need for the new topic. The inventory should include the range from simpler skills and concepts to the most difficult, in order to pinpoint those for which reteaching may be necessary.

Daily review of homework alone is not sufficient; it often concerns only a small portion of the needed prerequisites for a new topic.

A more extensive overview is needed at regular intervals. Thus, Good and Grouws (1979) propose both daily, weekly, and monthly reviews as part of their model for teaching mathematics. The daily reviews concern the concepts and skills associated with the previous day's lesson and the homework, which should include one or two review problems from earlier work. The weekly reviews focus on skills and concepts covered during the previous week, while the monthly reviews focus on skills and concepts covered since the last monthly review. The aim is to help the students develop a feeling of continuity about the mathematics they are learning, to help them reorganize the material at their own comprehension levels; and to provide systematic practice which will promote retention. Good and Grouws have shown that this promotes achievement.

After a topic or unit is taught, key points or objectives should be reviewed. Students thus become aware of the major highlights of the lesson, so they can focus on the mathematical skills or concepts that will be needed in future lessons. It should be made clear to students that this is not simply a collection of exercises and problems: the review includes those topics which are the most important to remember.

Long-term retention is best served if assignments about a particular skill are spread out in time, rather than concentrated within a short interval. Review immediately after instruction consolidates the ideas from that instruction, while delayed review aids in the relearning of forgotten material. One researcher (Gay, 1973) worked with eighth graders on four principles from algebra and geometry. She found that, if there were to be two periods of intensive review, it was better to space them at one and seven days after instruction, rather than on the first two days after the topic was taught, or after an interval of 6 and 7 days.

Additionally, research has indicated that it is better to have short periods of intensive review rather than long periods. Interspersing review throughout the textbook or curriculum is better than having a lot of review at one time.

Types of Review

A number of review procedures have been found by teachers and researchers to be effective. Some of the different types explored are discussed in this section.

Outlining. The process of making an outline forces students to organize ideas and provides a structure that will help students to put the ideas together. When students use the outline they have made as an aid in reviewing, restructuring or recall of the mathematical ideas is promoted.

Questioning. Several researchers have focused on review questions, finding them effective for revealing content that has not been meaningful to students. Burns (1960) studied whether thought-provoking review study questions, spaced throughout instruction in grade 6, would result in greater achievement with fractions and decimals. Examples of the questions he used are found in Figure 1.

How would you write a fraction that tells you something is divided into eight equal parts?

How can $7 \frac{1}{4}$ be changed to $6 \frac{5}{4}$?

Do you have to change $2 \frac{3}{5}$ to an improper fraction in order to find the product of $3 \frac{3}{5} \times 5$?

How can you know, before dividing, that the quotient to $7/8 \div 2/3$ is greater than 1?

How would you explain why .2 and .20 and .200 have the same value?

What checks can you use to find out whether the decimal point in the product for $4 \times .32$ is correctly placed?

How would you state the meaning of $2 \div .2$ in at least two ways?

Figure 1. Review study questions (Burns, 1930).

Students who used the review questions scored significantly higher than those using textbook drill pages. Pupils' reactions were good; they reported that they found the questions useful and interesting.

In another study on review procedures, Lee (1980) explored the effects of different types of review questions in a mathematics textbook. Seventh-grade students were given work on module-seven arithmetic. A review passage consisting of a summary of rules and examples was placed immediately after each original passage. A review question, consisting of either words or calculations, was then given (see Figure 2).

Word-type review question:

Let's think about the 4-minute clock in terms of the seasons. Spring can be assigned as 4-minute clock number "0", summer as "1", fall "2", and winter "3". Use these numbers and seasons to answer the following 4-minute clock problems:

- (a) If I went to New York in spring and had been in Texas 3 seasons earlier, in which season did I go to Texas? Write this problem in equation form and solve it.

Calculation-type review question:

Answer the following 7-minute clock multiplication problems.

$$2 \times 3 = \underline{\quad\quad\quad} \quad 6 \times 2 = \underline{\quad\quad\quad} \quad 3 \times 5 = \underline{\quad\quad\quad}$$

Figure 2. Types of review questions (Lee, 1980).

Review questions were found to facilitate retention by promoting comprehension. Feedback helped students to consolidate what they had learned in the course of answering the review questions, and they developed their ability to transfer problem-solving skills. The word-type review questions required a thorough understanding of the concepts and rules and the ability to apply them to new situations. On the other hand, the calculation-type review questions required only comprehension of a narrow range of concepts and rules, often focused on rote learning.

When review questions were added to review passages, inattentive reading often resulted. Students perceived this as too much reading; they felt they had relearned from the passage and therefore became bored with answering the questions. Thus, Lee concluded, only one type of review, passage or questions, should be used at a time, and if only one is used, it should probably be review questions, since they aided learning more.

Testing. A study with students in grades 6, 8, 10, 11, and 13 sought to determine if the learning and retention of mathematical knowledge was affected by any of three types of review procedures: testing, testing-with-explanation, and unit review (Clayton, 1974). For the unit review, a list of behavioral objectives for the topics taught was given to students, with a worked example and an optional exercise identical to the corresponding test item. The test group was only given a test and then the answers. Both the unit review and testing-with-explanation groups were given worked-out solutions for test items and optional exercises. The reviews enhanced the learning and retention of mathematics, with testing-with-explanation the most promising and testing alone the least promising. As in other studies, the usefulness of feedback in promoting achievement was apparent.

Group Size. The effect of small group and individual review on achievement in grade 6 were compared by Pence (1974). Achievement trends favored the small-group review, although not significantly, indicating that review could be conducted either individually or in small groups.

Homework Content. The role of differing types of homework designed for review in first-year algebra classes was explored by Friesen (1976). One group had homework consisting of exercises relating to the topic taught that day. Another group was assigned

fewer exercises relating to the topic taught that day, plus exploratory exercises on content taught each of the prior two days and review exercises on the first and third days after the teaching of a topic. The group having exploratory and review exercises achieved and retained better than the group having exercises related only to the daily topic.

Difficulty Level. What is the effect of level of difficulty? Summerfield (1975) gave fifth and sixth graders a brief review and an extensive review inserted into instructional materials on factors and primes. No significant differences were found. However, high scores on the achievement test made it appear that the review material was too easy. It may be that review is only helpful for those who need review. This fits with Gay's finding that it was better for students to choose the amount of review they were to have, rather than giving everyone the same amount.

Games. Bright et al. (1980) found that games provide effective review. In a motivating situation, students can focus on the mathematics and enjoy the process of recalling and restructuring. For instance, a game such as a variation of "Concentration", where students must match example with solution, can provide excellent review of their abilities with computation, definitions, or a variety of other topics.

Conclusion

Review is an important component of the mathematics instructional program. It can't be neglected — and it can be made interesting as well as profitable!

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